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Equitable Domination in Fuzzy Graphs

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ABSTRACT
In this paper we introduce the concepts of Equitable Domination in Fuzzy graphs .We determine the domination number γ_f and the equitable domination number γ_{fe}

***Keywords* –** Fuzzy graph, Fuzzy dominating set, fuzzy domination number, Fuzzy equitable dominating set and equitable domination number.

I. INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975.Though it is very young, it has been growing fast and the numerous applications in various fields. The first definition of fuzzy graphs was proposed by Kaufmann from the fuzzy relations introduced by Zadeh.Although Rosenfeld introduced another elaborated definition, including fuzzy vertex and fuzzy edges, several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness etc.The concept of domination in fuzzy graphs was investigated by A. Somasundramand S.Somasundram .A.Somasundaram presented the concepts of independent domination, total domination, connected domination and domination in Cartesian product and composition of fuzzy graphs.We review briefly some definitions in fuzzy graphs and introducesomenew notations. Prof. E. Sampath kumar is the first person to recognize the sprit and power of this concept and introduced various types of equitability in graphs like degree equitability, out wardequitability, inward equitability

Definition: 1.1

Let V be a finite non empty set. Let E be the collection of all two element subsets of V. A fuzzy graph G=$(\sigma,\mu)$is a set with two functions σ:V→ $[0,1]$ such that $\mu : V \times V \rightarrow [0,1]$such that $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$here after we write $\mu(x,y) = \mu(xy)$

Definition: 1.2
The order p and size q of a fuzzy graph G are defined to be p=$\sum_{u \in V} \sigma(u)$ and q= $\sum_{uv \in E} \mu(uv)$

Definition: 1.2

The strength of the connectedness between two nodes u, v in a fuzzy graph G is

$$\mu^{\infty}(u,v) = \sup\{\mu^{k}(u,v) : k = 1,2,3\cdots\cdots\}$$

Where $\mu^k(u,v) =$

$$\sup\left\{\begin{matrix}\mu(u,u_1) \wedge \mu(u_1,u_2) \wedge \mu(u_2,u_3) \wedge \cdots\cdots \\ \mu(u_{k-1},v)\end{matrix}\right\}$$

Definition: 1.3

An arc (u, v) is said to be a strong arc or strong edge if $\mu(u,v) \geq \mu^{\infty}(u,v)$ and the node v is said to be strong neighbor of u.If (u, v) is not a strong arc then v is called isolated node or isolated vertex.

Definition: 1.4

In a fuzzy graph every arc is strong arc then the fuzzy graph is called strong arc fuzzy graph.

Definition: 1.5

An edge e=xy of a fuzzy graph is called an effective edge if $\mu(x,y) = \sigma(x) \wedge \sigma(y)$.

Let u be a node in fuzzy graph G then N (u) = {v :(u, v) is strong arc} is called neighborhood of u and N[u] =N (u)$\cup \{u\}$is called closed neighborhood of u.

Definition: 1.6

Let G= (σ,μ) be a fuzzy graph .A subset S of V is called a dominating set in G if every vertex in V-S, there exists u$\in S$ such that u dominates v. The domination number of G is the minimum cardinality taken over all dominating sets in G and is denoted by $\gamma(G)$ or simply γ_f. A fuzzy dominating set S of a fuzzy graph G is called minimal fuzzy dominating set of G, if for every node v$\in S$, S-{v} is not a fuzzy dominating set.

Definition: 1.7

Let G:(σ,μ) be a fuzzy graph .The degree of a vertex v is $d_G(v)=\sum_{u\neq v}\mu(u,v)$.Since $\mu(u,v)>$ 0 for uv$\in E$ and $\mu(uv)=0$ *for* $uv\notin E$ this is equivalent to $d_G(v)=\sum_{uv\varepsilon E}\mu(uv)$.The minimum degree of G is $\delta(G)=\wedge\{d(v)/v\varepsilon V\}$.The maximum degree of G is $\Delta(G)=\vee\{d(v)/v\varepsilon V\}$.

Let G:(σ,μ) be a fuzzy graph on $G^*:(V,E)$.If $d_G(v)$=K for all v ε V.

(i.e.) if each vertex has same degree K,then G is said to be a regular fuzzy graph of degree K or a K-regular fuzzy graph.

II. MAIN RESULTS

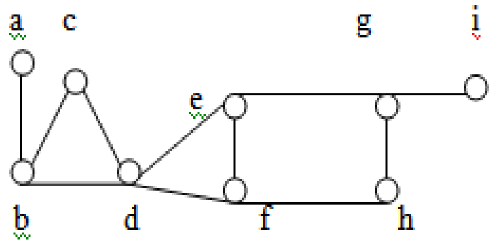
Definition: 2.1

A subset S of V is called an fuzzy equitable dominating set if for every v ε V-S there exists a vertex uε S such that uv$\in$E(G) and $|\deg(u)-\deg(v)|\leq 1$.The minimum cardinality of such a dominating set is denoted by γ_{fe} and is called the equitable domination number of G.

Definition: 2.2

A fuzzy equitable dominating set S is said to be a minimal equitable dominating set if no proper subset of S is an equitable dominating set.

EXAMPLE: 2.3



$\sigma(a)$=0.4 , $\sigma(b)$=0.3, $\sigma(c)$=0.1, $\sigma(d)$=0.2
$\sigma(e)$=0.4, $\sigma(f)$=0.5, $\sigma(g)$=0.3, $\sigma(h)$=0.3
$\sigma(i)$=0.4, $\mu(a,b)$=0.2, $\mu(b,c)$=0.1, $\mu(b,d)$=0.2, $\mu(c,d)$=0.1, $\mu(d,e)$=0.2, $\mu(d,f)$=0.2, $\mu(e,f)$=0.4, $\mu(e,g)$=0.3, $\mu(f,h)$=0.3, $\mu(g,h)$=0.3, $\mu(g,i)$=0.3,

Theorem: 2.4

An fuzzy equitable dominating set S is minimal if and only if for every vertex uε S one of the
Following holds.

(i) Either N (u) $\cap S=\phi$

(ii) There exists a vertex vε V-S such that N (v) $\cap$ s = {u} and $|\deg(v)-\deg(u)|\leq 1$.

Proof:

Assume that S is a minimal fuzzy equitable dominating set. Suppose (i) and (ii) does not hold. Then for some uεS there exists vε $N(u)\cap S$ such that $|\deg(v)-\deg(u)|\leq 1$.and for every vε V-S either $N(v)\cap S\neq\{u\}$ or $|\deg(v)-\deg(u)|\geq 2$ or both.Therfore S-{u} is an fuzzy equitable dominating set, which is a contradiction to the minimality of S.Therfore (i) and (ii) holds. Conversely suppose for every u$\in S$, one of the statements (i) and (ii) holds. Suppose S is not minimal. Then there exists u$\in S$ such that S-{u} is an fuzzy equitable dominating set. Therefore there exists v$\in S-\{u\}$ such that v fuzzy equitably dominates u, that is v$\in N(u)$ and $|\deg(v)-\deg(u)|\leq 1$. Therefore u does not satisfy (i). Then u must satisfy (ii). Then there exists a $v\in V-S$ Such that $N(v)\cap S=\{u\}$ and $|\deg(v)-\deg(u)|\leq 1$. Since S-{u} is a fuzzy equitable dominating set, there exists $w\in S-\{u\}$ such that w is adjacent to v and w is degree fuzzy equitable with v.

$$\therefore w\in N(v)\cap S\ ,|\deg(w)-\deg(v)|\leq 1\ \ and$$

$$w_0\neq u$$

$$which\ is\ a\ contradiction\ \ to\ N(v)\cap S=\{u\}$$

Therefore S is a minimal fuzzy equitable dominating set.

THEOREM: 2.5

If G is a regular fuzzy graph or (r, r+1) biregular fuzzy graph for some fuzzy graph, for some r, then

$$\gamma_{fe}=\gamma_f$$

Proof:

Suppose G is a regular fuzzy graph .Then every vertex of G has the same degree say r.Let S be a minimum dominating set of G .Then $|S|=\gamma(G)=\gamma_f$ .Let u $u\in V-S$ then as S is a dominating set, thert exists

$v \in S$ and $uv \in E(G)$.Also deg (u) =deg (v) =r.

$\left|\deg(u) - \deg(v)\right| = 0 \leq 1$

$\therefore$ S is an fuzzy equitable set of G so that $\gamma_{fe}(G) \leq |S| = \gamma_f$ *But* $\gamma_f \leq \gamma_{fe}$

$\therefore \lambda_f = \gamma_{fe}$

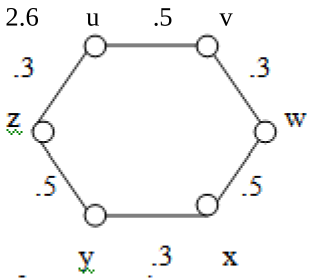
Suppose G is a biregular fuzzy graph then for every vertex of G has degree either r(or) r+1.Let S be a minimum dominating set of G .Then $|s| = \gamma_f$ .Let $u \in$ V-S,then as S is a dominating set there exists $v \in$ S,and$uv \in$ E(G).Also deg(u)=r (or) r+1and deg(v)=r(or)r+1.Therefore $\left|\deg(u) - \deg(v)\right| = 1$

$\therefore$ S is a fuzzy equitable dominating set of G

$\therefore \gamma_{fe} \leq |S| = \gamma_f$ *But*$\gamma_f \leq \gamma_{fe}$

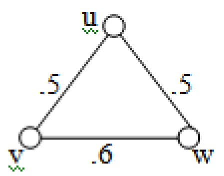
$\therefore \gamma_f = \gamma_{fe}$

EXAMPLE: 2.6



G is a regular fuzzy graph
S= {a, d}
V-S= {b, c, e, f} and S is an fuzzy equitable dominating set

EXAMPLE: 2.7



G is a bi-regular fuzzy graph.

Remark

Since any fuzzy equitable dominating set is also a dominating set,$\therefore \lambda_f \leq \gamma_{fe}$ for any graph G.

Remark:

If a vertex $u \in V$ be such that $\left|\deg(u) - \deg(v)\right| \geq 2$ for all $v \in$ N(u) then u is in every fuzzy equitable dominating set and the points are called equitable isolates. Let I_{fe} denote the set of all fuzzy equitable isolates.

Theorem 2.9:

A fuzzy graph G has a unique minimal fuzzy equitable dominating set if and only if the set of all fuzzy equitable isolates forms an equitable dominating set.

Proof

Sufficient conditionis obvious. Let G have a unique minimal fuzzy equitable dominating set S.
Let D= {$u \in$ V/u is a fuzzy equitable isolate.Then $D \subseteq S$ .We shall prove that D=S

Suppose S-D$\neq \phi$.Let $v \in$ S-D since v is not a fuzzy equitable isolate, V-{v} is a fuzzy equitable dominating set. Hence there exists a minimal fuzzy equitable dominating set. $S_1 \subseteq V - \{v\}$ and $S_1 \neq S$ a contradicts to the fact that G has a unique fuzzy equitable dominating set.

Therefore D=S.

III. FUZZY EQUITABLE INDEPENDENT SETS:

Definition: 3.1

Two nodes of a fuzzy graph are said to be fuzzy independent if there is no strong arc between them .A subset S of V is said to be a fuzzy independent set of G if any two nodes of S are fuzzy independent.

Definition: 3.2

Let $u \in$ V the fuzzy equitable neighborhood of u denoted by $N^{fe}(u)$ is defined as

$N^{fe}(u)$= {$v \in$ V/$v \in$ N (u), u, v is a strong arc and$\left|\deg(u) - \deg(v)\right| \leq 1$}and

$u \in I_{fe} \Leftrightarrow N^{fe}(u) = \phi$.

The cardinality of $N^{fe}(u)$ is denoted by $\deg_G{}^{fe}(u)$

Definition: 3.3

The maximum and minimum fuzzy equitable degree of a point in G are denoted by $\Delta^{fe}(G)$ *and* $\delta^{fe}(G)$.That is

$\Delta^{fe}(G) = \max_{u \in V(|G|)} \left|N^{fe}(u)\right|$ *and*

$\delta^{fe}(G) = \min_{u \in V(|G|)} \left|N^{fe}(u)\right|$

Definition: 3.4

A subset S of V is called a fuzzy equitable independent set if for any $u \in$ S, $v \notin N^{fe}(u)$ for all $v \in S - \{u\}$

Theorem: 3.5

Let S be a maximal fuzzy equitable independent set. Then S is a minimal fuzzy equitable dominating set.

Proof:

Let S be a maximal fuzzy equitable independent set. Let u $\in$ V-S .If u $\notin N^e(v)$ for every v $\in$ S, then S $\cup$ {u} is an fuzzy equitable independent set, a contradiction to the maximality of S .Therefore u $\in N^e(v)$ for some v $\in$ S .Therefore S is an fuzzy equitable dominating set.Since for any u $\in$ S , u $\notin N^e(v)$ for every v $\in$ S-{u},either N(u) $\cap S = \phi$ or $|\deg(v) - \deg(u)| \geq 2$ for all v $\in N(u) \cap S$ .Therefore S is a minimal fuzzy equitable dominating set.

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